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The Piezo-Electric Effect in Nematic Layers

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This paper discusses the influence of piezo-electricity on deformation of nematic liquids in an electric field.

The deformation of nematic liquid crystals in a magnetic field has first been calculated by Saupe.¹ The analogous problem for the electric field is more complicated due to the large dielectric anisotropy of many nematic liquids. This problem was solved independently by Deuling² and by Gruler, Scheffer and Meier.³ Both papers, however, did not include the deformation polarization or piezo-electric effects. As pointed out first by R. B. Meyer,⁴ a nematic liquid may show an electric polarisation \vec{P} in response to mechanical deformation. In the present paper we calculate the deformation of a nematic liquid layer in an electric field including the piezo-electric effect.

For definiteness let us assume a nematic liquid layer enclosed between two parallel metalized glass plates of distance L with the molecules orientated parallel to the plates. Applying a voltage U to the plates will cause the molecules to tilt at an angle φ which is a function of coordinate z . The function $\varphi(z)$ is zero for $z = 0$ and $z = L$ and assumes a maximum φ_m in the middle. We can calculate φ as a function of z by minimizing the free energy. The free energy per unit area is

$$G = \frac{1}{2} \int_0^L (k_{11} \cos^2 \varphi + k_{33} \sin^2 \varphi) \left(\frac{d\varphi}{dz} \right)^2 dz - \frac{1}{2} \int_0^L \epsilon_0 \vec{E} \cdot \epsilon \cdot \vec{E} dz - \int_0^L \vec{P} \cdot \vec{E} dz \quad (1)$$

where \vec{E} is the electric field, \vec{D} the dielectric displacement and \vec{P} the piezo electric polarization. Adopting the notation of Meyer⁴ we have for \vec{P}

$$\vec{P} = e_{11} \vec{n} \operatorname{div} \vec{n} - e_{33} \vec{n} \times \operatorname{rot} \vec{n} \quad (2)$$

the vector \vec{n} being the director and e_{11} , e_{33} being the piezo-electric coefficients. Helfrich⁵ has estimated these coefficients to be of the order of $4 \cdot 10^{-5}$ cgs units. Schmidt, Schadt and Helfrich⁶ found $e_{33} = 3.7 \cdot 10^{-5}$ cgs units for MBBA. We can rewrite (1) in the more convenient form

$$G = \frac{1}{2} \int_0^L (k_{11} \cos^2 \phi + k_{33} \sin^2 \phi) \left(\frac{d\phi}{dz} \right)^2 dz - \frac{1}{2} U \cdot D_z - \frac{1}{2} \int_0^L P_z \cdot E_z dz \quad (3)$$

For D_z we have

$$D_z = \epsilon_0 E_z (\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi) + (e_{11} + e_{33}) \cos \phi \sin \phi \frac{d\phi}{dz} \quad (4)$$

From this expression one finds the last term in (3) to be

$$- \frac{1}{2} \int_0^L P_z E_z dz = \frac{1}{2} \frac{(e_{11} + e_{33})^2}{\epsilon_0} \int_0^L \frac{\cos^2 \phi \sin^2 \phi}{\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi} \left(\frac{d\phi}{dz} \right)^2 dz \quad (5)$$

The free energy now is

$$G = \frac{1}{2} \int_0^L \left\{ k_{11} \cos^2 \phi + k_{33} \sin^2 \phi + \frac{(e_{11} + e_{33})^2}{\epsilon_0} \frac{\cos^2 \phi \sin^2 \phi}{\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi} \right\} \left(\frac{d\phi}{dz} \right)^2 dz - \frac{1}{2} U \cdot D_z \quad (5)$$

From (4) we find D_z as functional of $\phi(z)$

$$D_z = \frac{\epsilon_0 U}{\int_0^L (\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi)^{-1} dz} \quad (6)$$

The calculation proceeds now exactly as in ref. (2) to give the differential equation

$$\left\{ k_{11} \cos^2 \phi + k_{33} \sin^2 \phi + \frac{(e_{11} + e_{33})^2}{\epsilon_0} \frac{\cos^2 \phi \sin^2 \phi}{\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi} \right\} \left(\frac{d\phi}{dz} \right)^2 =$$

$$= C + \frac{D_z^2 / \epsilon_0}{\epsilon_{\parallel} \sin^2 \phi + \epsilon_{\perp} \cos^2 \phi} \quad (7)$$

The constant of integration C is determined by the condition $d\phi/dz = 0$ and $\phi = \phi_m$ for $z = L/2$. Introducing constants

$$\gamma = (\epsilon_{\parallel} - \epsilon_{\perp})/\epsilon_{\perp}; \quad \kappa = (k_{33} - k_{11})/k_{11}; \quad \Delta = \frac{(e_{11} + e_{33})^2}{k_{11} \epsilon_{\perp}}; \quad (8)$$

$$\nu = (n_e^2 - n_o^2) / n_o^2$$

we can write

$$\frac{d\phi}{dz} = D_z \frac{\sqrt{\gamma/(\epsilon_0 \epsilon_{\perp} k_{11})}}{\sqrt{1 + \gamma \sin^2 \phi_m}} \left\{ \frac{\sin^2 \phi_m - \sin^2 \phi}{(1 + \kappa \sin^2 \phi)(1 + \gamma \sin^2 \phi) + \Delta \cos^2 \phi \sin^2 \phi} \right\}^{1/2} \quad (9)$$

From equation (6) we can find the voltage U with the aid of (9)

$$U/U_0 = \frac{2}{\pi} \sqrt{1 + \gamma \sin^2 \phi_m} \int_0^{\phi_m} \left\{ \frac{(1 + \kappa \sin^2 \phi)(1 + \gamma \sin^2 \phi) + \Delta \cos^2 \phi \sin^2 \phi}{(\sin^2 \phi_m - \sin^2 \phi)(1 + \gamma \sin^2 \phi)^2} \right\}^{1/2} dz \quad (10)$$

U_0 is a natural unit of voltage²

$$U_0 = \pi \left(\frac{k_{11}}{\epsilon_0 (\epsilon_{\parallel} - \epsilon_{\perp})} \right)^{1/2} \quad (11)$$

From this expression for U/U_0 we can find the maximum angle of deformation for any given value of voltage U . One can observe the deformation by measuring the optical phase difference d between light polarized in the y -direction and the x -direction.

$$d = \frac{1}{\lambda} \int_0^L (n_o - n(z)) dz \quad (12)$$

where λ is the wavelength of the light and the index of refraction $n(z)$ is given by

$$n(z) = n_e n_o / \sqrt{n_e^2 \sin^2 \phi + n_o^2 \cos^2 \phi} \quad (13)$$

For the quantity $\delta = d(U) - d(0)$ one gets the equation

$$\begin{aligned} \left(1 - \frac{\delta \cdot \lambda}{n_e \cdot L}\right) \cdot \int_0^{\phi_m} \left\{ \frac{(1 + \kappa \sin^2 \phi)(1 + \gamma \sin^2 \phi) + \Delta \cos^2 \phi \sin^2 \phi}{\sin^2 \phi_m - \sin^2 \phi} \right\}^{\frac{1}{2}} d\phi \\ = \int_0^{\phi_m} \left\{ \frac{(1 + \kappa \sin^2 \phi)(1 + \gamma \sin^2 \phi) + \Delta \cos^2 \phi \sin^2 \phi}{(1 + \nu \sin^2 \phi)(\sin^2 \phi_m - \sin^2 \phi)} \right\}^{\frac{1}{2}} d\phi \end{aligned} \quad (14)$$

The expressions (10) and (14) differ from the corresponding expressions of ref. (2) only by the term $\Delta \cos^2 \phi \sin^2 \phi$ describing the influence of piezo electricity on the deformation. Using the estimate of Helfrich⁵ for e_{11} and e_{33} one finds Δ to be of the order of 0.1.

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