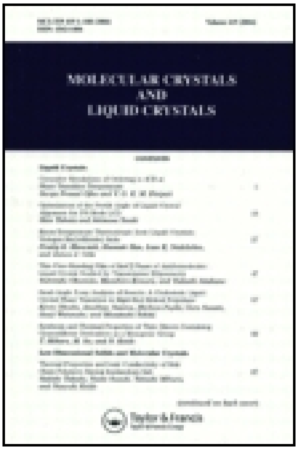

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The Piezo-Electric Effect in Nematic Layers

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The Piezo-Electric Effect in Nematic Layers

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This paper discusses the influence of piezo-electricity on deformation of nematic liquids in an electric field.

The deformation of nematic liquid crystals in a magnetic field has first been calculated by Saupe.[1] The analogous problem for the electric field is more complicated due to the large dielectric anisotropy of many nematic liquids. This problem was solved independently by Deuling[2] and by Gruler, Scheffer and Meier.[3] Both papers, however, did not include the deformation polarization or piezo-electric effects. As pointed out first by R. B. Meyer,[4] a nematic liquid may show an electric polarisation $\vec{P}$ in response to mechanical deformation. In the present paper we calculate the deformation of a nematic liquid layer in an electric field including the piezo-electric effect.

For defineteness let us assume a nematic liquid layer enclosed between two parallel metalized glass plates of distance L with the molecules orientated parallel to the plates. Applying a voltage U to the plates will cause the molecules to tilt at an angle φ which is a function of coordinate z. The function φ (z) is zero for z = o and z = L and assumes a maximum φ_m in the middle. We can calculate φ as a function of z by minimizing the free energy. The free energy per unit area is

$$G = \frac{1}{2} \int_0^L (k_{11} \cos^2\phi + k_{33} \sin^2\phi) \left(\frac{d\phi}{dz}\right)^2 dz - \\ - \frac{1}{2} \int_0^L \epsilon_0 \vec{E}\cdot\epsilon\cdot\vec{E}\, dz - \int_0^L \vec{P}\cdot\vec{E}\, dz \qquad (1)$$

where $\vec{E}$ is the electric field, $\vec{D}$ the dielectric displacement and $\vec{P}$ the piezo electric polarization. Adopting the notation of Meyer[4] we have for $\vec{P}$

$$\vec{P} = e_{11}\ \vec{n}\ \mathrm{div}\,\vec{n} - e_{33}\ \vec{n} \times \mathrm{rot}\,\vec{n} \tag{2}$$

the vector $\vec{n}$ being the director and e_{11}, e_{33} being the piezo-electric coefficients. Helfrich[5] has estimated these coefficients to be of the order of $4 \cdot 10^{-5}$ cgs units. Schmidt, Schadt and Helfrich[6] found $e_{33} = 3.7 \cdot 10^{-5}$ cgs units for MBBA. We can rewrite (1) in the more convenient form

$$G = \frac{1}{2} \int_0^L (k_{11} \cos^2\phi + k_{33} \sin^2\phi) \left(\frac{d\phi}{dz}\right)^2 \cdot dz - \frac{1}{2}\ U \cdot D_z - \frac{1}{2} \int_0^L P_z \cdot E_z\, dz \tag{3}$$

For D_z we have

$$D_z = \epsilon_0 E_z (\epsilon_\| \sin^2\phi + \epsilon_\perp \cos^2\phi) + (e_{11} + e_{33}) \cos\phi \sin\phi \frac{d\phi}{dz} \tag{4}$$

From this expression one finds the last term in (3) to be

$$-\frac{1}{2} \int_0^L P_z E_z dz = \frac{1}{2} \frac{(e_{11} + e_{33})^2}{\epsilon_0} \int_0^L \frac{\cos^2\varphi \sin^2\varphi}{\epsilon_\| \sin^2\varphi + \epsilon_\perp \cos^2\varphi} \left(\frac{d\varphi}{dz}\right)^2 dz \tag{5}$$

The free energy now is

$$G = \frac{1}{2} \int_0^L \left\{k_{11} \cos^2\phi + k_{33} \sin^2\phi + \frac{(e_{11} + e_{33})^2}{\epsilon_0} \frac{\cos^2\phi \sin^2\phi}{\epsilon_\| \sin^2\phi + \epsilon_\perp \cos^2\phi}\right\} \left(\frac{d\phi}{dz}\right)^2 dz - \frac{1}{2} U \cdot D_z \tag{5}$$

From (4) we find D_z as functional of $\varphi(z)$

$$D_z = \epsilon_0 U \Big/ \int_0^L (\epsilon_\| \sin^2\phi + \epsilon_\perp \cos^2\phi)^{-1}\, dz \tag{6}$$

The calculation proceeds now exactly as in ref. (2) to give the differential equation

$$\left\{k_{11}\cos^2\phi + k_{33}\sin^2\phi + \frac{(e_{11}+e_{33})^2}{\epsilon_0}\;\frac{\cos^2\phi\sin^2\phi}{\epsilon_{\|}\sin^2\phi + \epsilon_{\perp}\cos^2\phi}\right\}\left(\frac{d\phi}{dz}\right)^2 =$$

$$= C + \frac{D_z^2/\epsilon_0}{\epsilon_{\|}\sin^2\phi + \epsilon_{\perp}\cos^2\phi} \qquad (7)$$

The constant of integration C is determined by the condition $d\varphi/dz = 0$ and $\varphi = \varphi_m$ for $z = L/2$. Introducing constants

$$\gamma = (\epsilon_{\|} - \epsilon_{\perp})/\epsilon_{\perp}\,;\quad \kappa = (k_{33} - k_{11})/k_{11}\,;\quad \Delta = \frac{(e_{11}+e_{33})^2}{k_{11}\,\epsilon_{\perp}}\,;$$
$$\nu = (n_e^2 - n_o^2)/n_0^2 \qquad (8)$$

we can write

$$\frac{d\phi}{dz} = D_z\,\frac{\sqrt{\gamma/(\epsilon_0\epsilon_{\perp}k_{11})}}{\sqrt{1+\gamma\sin^2\phi_m}}\left\{\frac{\sin^2\phi_m - \sin^2\phi}{(1+\kappa\sin^2\phi)(1+\gamma\sin^2\phi) + \Delta\cos^2\phi\sin^2\phi}\right\}^{1/2} \qquad (9)$$

From equation (6) we can find the voltage U with the aid of (9)

$$U/U_0 = \frac{2}{\pi}\sqrt{1+\gamma\sin^2\phi_m}\int_0^{\phi m}\left\{\frac{(1+\kappa\sin^2\phi)(1+\gamma\sin^2\phi) + \Delta\cos^2\phi\sin^2\phi}{(\sin^2\phi_m - \sin^2\phi)(1+\gamma\sin^2\phi)^2}\right\}^{1/2} dz \qquad (10)$$

U_0 is a natural unit of voltage[2]

$$U_0 = \pi\left(\frac{k_{11}}{\epsilon_0(\epsilon_{\|} - \epsilon_{\perp})}\right)^{1/2} \qquad (11)$$

From this expression for U/U_0 we can find the maximum angle of deformation for any given value of voltage U. One can observe the deformation by measuring the optical phase difference d between light polarized in the y-direction and the x-direction.

$$d = \frac{1}{\lambda}\int_0^L (n_0 - n(z))\;dz \qquad (12)$$

where λ is the wavelength of the light and the index of refraction $n(z)$ is given by

$$n(z) = n_e\, n_o \,/\, \sqrt{n_e^2 \sin^2\phi + n_0^2 \cos^2\phi} \tag{13}$$

For the quantity $\delta = d(U) - d(0)$ one gets the equation

$$\left(1 - \frac{\delta \cdot \lambda}{n_e \cdot L}\right) \cdot \int_0^{\phi m} \left\{ \frac{(1+\kappa \sin^2\phi)(1+\gamma \sin^2\phi) + \Delta \cos^2\phi \sin^2\phi}{\sin^2\phi_m - \sin^2\phi} \right\}^{1/2} d\phi$$

$$= \int \left\{ \frac{(1+\kappa \sin^2\phi)(1+\gamma \sin^2\phi) + \Delta \cos^2\phi \sin^2\phi}{(1+\nu \sin^2\phi)(\sin^2\phi_m - \sin^2\phi)} \right\}^{1/2} d\phi \tag{14}$$

The expressions (10) and (14) differ from the corresponding expressions of ref. (2) only by the term $\Delta \cos^2\varphi \sin^2\varphi$ describing the influence of piezo electricity on the deformation. Using the estimate of Helfrich[5] for e_{11} and e_{33} one finds Δ to be of the order of 0.1.

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